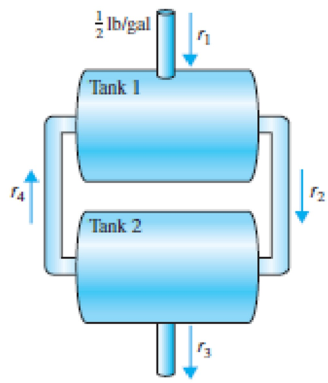
Two Tank System

annotate

A two Tank System

Consider the two-tank mixing apparatus shown in the figure. Each tank has a capacity of 500 gal and initially contains 100 gal of fresh water. At time $t = 0$, the well-stirred mixing process begins with the specified input concentration and flow rates.



Volume
Tank 1: V_1
Tank 2: V_2

Salt
Tank 1: Q_1
Tank 2: Q_2

concentration
Tank 1: C_1
Tank 2: C_2

Max Capacity: 500 gal

$V_1(0) = 100$ gal

$V_2(0) = 100$ gal

$Q_1(0) = 0$

$Q_2(0) = 0$

$C(t) = \dfrac{Q(t)}{V(t)}$

$$Q_1'(t) = C_{i_1} r_{i_1} - C_{o_1} r_{o_1} = \underbrace{\tfrac{1}{2} r_{i_1} + \boxed{C_{i_2}(t)}\, r_4}_{\text{into Tank 1}} - \underbrace{\boxed{C_{i_1}(t)}\, r_2}_{\text{out of Tank 1}}$$

$C_{o_2}(t) = \dfrac{Q_2(t)}{V_2(t)}$ $\qquad$ $C_{o_1}(t) = \dfrac{Q_1(t)}{V_1(t)}$

$$Q_2'(t) = C_{i_2} r_{i_2} - C_{o_2} r_{o_2} = \underbrace{r_2 \frac{Q_1(t)}{V_1(t)}}_{\text{into Tank 1}} - \underbrace{\frac{Q_2(t)}{V_2(t)}(r_3 + r_4)}_{\text{out of Tank 2}}$$

$V_1(t) = V_1(0) + (r_{i_1} + r_{o_1})t = 100 + (r_1 + r_4 - r_2)t$

$V_2(t) = V_2(0) + (r_{i_2} + r_{o_2})t = 100 + (r_2 - (r_3 + r_4))t$

Problem 30

$r_1 = r_3 = 5$ gal/min

$r_2 = 6$ gal/min

$r_4 = 4$ gal/min

$V_2(t) = 100 + (6 - 9)t = 100 - 3t = 0 \Rightarrow t_2^* = \frac{100}{3} \approx 33.\overline{3}$ min

$V_1(t) = 100 + (9 - 6)t = 100 + 3t = 500 \Rightarrow t_1^* = \frac{400}{3} \approx 1.5$ hour

$t^* = \min(t_1^*, t_2^*) \approx 1.5$ hr

$0 \le t \le t_2^*$ interval of interest

A Two-Tank Mixing Problem

Consider the two-tank connection shown in Figure 4.1. As in Chapter 2, the solute and solvent are assumed to be salt and water, respectively, and the solutions in both tanks are "well-stirred." Assume each tank has a capacity of 500 gal. Initially, Tank 1 contains 200 gal of fresh water, while Tank 2 has 50 lb of salt dissolved in 300 gal of water. At time $t = 0$, the flow begins at the rates and concentrations shown in Figure 4.1. Let the amounts of salt in the two tanks at time t be denoted by $Q_1(t)$ and $Q_2(t)$, respectively. The problem is to determine $Q_1(t)$ and $Q_2(t)$ on the time interval that is physically relevant (that is, we will stop the flow if one of the tanks becomes completely filled or completely drained). Time t is in minutes.

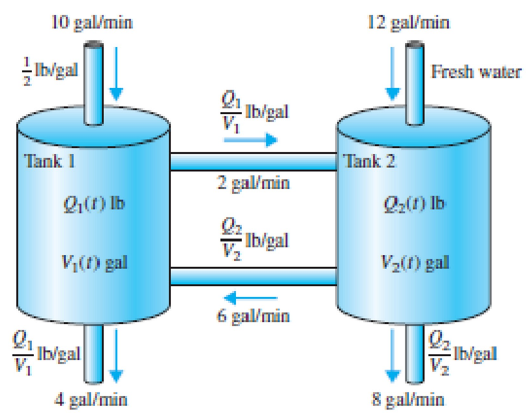


FIGURE 4.1

$V_1(0) = 200$ gal.

$Q_1(0) = 0$ - Fresh Water

$C_1(0) = 0$

$V_2(0) = 300$ gal.

$Q_2(0) = 50$ lb.

$C_1(0) = 1/6$?

$$C(t) = \frac{Q(t)}{V(t)}$$

$$Q_1'(t) = C_{i_1} r_{i_1} - C_{o_1} r_{o_1}$$
$$= \underbrace{\tfrac{1}{2}(10) + 6\left(\frac{Q_2(t)}{V_2(t)}\right)}_{\text{IN}} - \underbrace{(2+4)\frac{Q_1(t)}{V_1(t)}}_{\text{out}}$$

$$Q_2(t) = C_{i_2} r_{i_2} - C_{o_2} r_{o_2}$$
$$= \underbrace{\left(12(0) + 2\frac{Q_1(t)}{V_1(t)}\right)}_{\text{IN}} - \underbrace{\frac{Q_2(t)}{V_2(t)}(6+8)}_{\text{out}}$$

Volumes:

$$V_1(t) = 200 + (r_{i_1} - r_{o_1})t = 200 + (10+6-(2+4))t = 200 + 10t$$

$$V_2(t) = 300 + (r_{i_2} - r_{o_2})t = 300 + (12+2-(6+8))t = 300$$

when tank 1 becomes full (500 gal)

Time of interest $0 \le t \le t^* = t_1^* = 30$ min